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ATTENTION
MEMBERS OF L. T. A. MATHEMATICS SECTION !

In order that the Mathematics Section of the Louisiana Teachers Association may render the greatest possible service to the members I am asking for the cooperation of the teachers of mathematics of the state. I should like especially to have suggestions as to what subjects should be discussed at the meeting of the Section at the next convention. Suggestions as to who should be asked to appear on this program would be appreciated, also.

Respectfully,

W. C. ROATEN, Chairman,
Mathematics Section.

Oakdale, La.

NEW OFFICERS ELECTED AT THE OXFORD MATHEMATICAL MEET

At the joint annual meeting of the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics and the Louisiana-Mississippi Section of M. A. of A., held at Oxford, Mississippi, March 11 and 12, the following were elected as the new officials of the two organizations:

Officers of the Council are:

President, Professor Eugene Bigham, University Demonstration School, Oxford Miss.; Mississippi Vice-President, Professor C. E. Beckett, Central H. S., Jackson, Miss.; Louisiana Vice-President, Miss Dora Forno, New Orleans Normal School, New Orleans, La.; Secretary, Professor Henry Schroeder, L. P. I., Ruston, La.

Officers of the Section are:

President, Professor C. D. Smith, Mississippi State College, Starkville, Miss.; Mississippi Vice-President, Professor T. A. Bickerstaff, University, Miss.; Louisiana Vice-President, Dr. H. L. Smith, L. S. U., Baton Rouge, La.; Secretary, Professor May Hickey, Delta State Teachers College, Cleveland, Miss.

The next annual meeting of the two organizations will be held in Ruston, Louisiana. The 1934 meeting will be held again in Mississippi, Jackson having been selected as the meeting place.

THE MATHEMATICAL JOURNAL IN AMERICA

In the American Year Book of 1931, Professor Tomlinson Fort, who wrote the article entitled "*Mathematics*", refers to the crying need of more journals in America in order that more of the great amount of mathematical research being done can be published. Even as far back as 1916 Professor G. A. Miller in his "*Historical Introduction to Mathematical Literature*" wrote as follows, referring to a world mathematical situation rather than to the American one in particular: "It is a rather singular fact that when mathematical periodicals became more numerous and seemed to remove the greatest need of mathematical meetings, since they established improved means of communication between the various mathematical investigators, these meetings became more numerous and more popular. This seems to indicate that periodicals have been of the greatest value in arousing interest, for when men travel hundreds and even thousands of miles to attend mathematical meetings there must be considerable interest". Ten years earlier even than this, Cajori, in his History of Mathematics alluded to the productiveness of modern mathematical writers as being "enormous", while he quoted in the same connection words from Professor Cayley, namely, "It is difficult to give an idea of the extent of modern mathematics."

It is possible, indeed we think rather probable, that a true interpretation of the relation between a multiplied number of mathematical journals and large annual meetings of mathematical workers is, not that the latter is caused by the former, but, rather that both these phenomena are based on the same underlying cause, namely, a properly distributed public interest in mathematics, an interest which is by no means identical with mere research interest.

We like to think that there is such a class as a mathematical laity, much larger in number than the class of researchers, and, in many respects as fundamentally important for advancing the science. It is a class of intelligent, even scholarly, appreciators of mathematics and mathematical values. Forming part of this class may be even the potential research mind, with research promise more independent and self-reliant, possibly, than the type of ability that is rather prone to lean upon academic guidance. Such a mind is likely to experience self-discovery through the journals rather than the schools.

Since Cayley's remark, made 49 years ago, concerning the vast extent of modern mathematics, the rate of mathematical output has

increased enormously not only in Europe but in America as well. Indeed one may feel entirely safe in stating that the amount of mathematical product made in America since 1900 is incomparably greater than the volume of American mathematical product in all the time prior to 1900. Notwithstanding this fact it is true that from 1900 to 1925 throughout the whole of the United States not a single mathematical journal of collegiate or higher grade was established, the *American Journal of Mathematics* having been founded in 1878, *Annals of Mathematics* in 1899, *Bulletin of the American Mathematical Society* in 1894, *Transactions of the Society* in 1900, the *American Mathematical Monthly* in 1894. It should be observed that this statement does not conflict with one made by Professor R. C. Archibald in the October, 1931, issue of the *American Mathematical Monthly*, in an article entitled "New Mathematical Periodicals", namely, that three mathematical journals, including the Mathematics News Letter, have been established in the United States within the last five years.

This slowness of America in providing an adequate number of public carriers of our so rapidly increasing mathematical knowledge, in the form of circulating journals studiously adapted to stimulate the mathematical interest of the scholarly layman no less than to put him in contact with research material, is undoubtedly responsible, in a measure, for the growing attitude of aloofness toward mathematics on the part of much of the American public. Furthermore, this condition of things is seriously aggravated by the existence of certain powerful tendencies inherent in the very nature of mathematics. We refer to the fact, generally recognized, that the mathematics game may so very easily be played as a game of solitaire. It is true especially of the pure mathematician that about the only human contact necessary to him—occasionally may be found one disposed to deny even this—is contact with his fellow mathematicians. A consistent check just here would be another quotation from G. A. Miller's "Historical Introduction, etc.", "While many of the leading mathematicians have taken part in the national and international (mathematical) meetings, there are others of equal prominence who have seldom attended such meetings."

Thus, it is scarcely to be denied, that this exemption of mathematics from so much of the human contact necessary to the proper functioning of other sciences places it at a serious disadvantage when, competing with these other sciences and with the arts and the humanities, it seeks a wholesome degree of publicity or popular favor.

Thus, for this reason alone, if for no other, is there greater need for a journalized mathematical literature in order that its just and fair interests may be preserved. Today multiplied mathematical journals are more vital to a balanced mathematical program in America than are chemistry journals vital to American chemistry or than biological journals are vital to the American biological sciences.

We conclude our study with some statistical comparisons, our figures having been compiled with the aid of one of our Louisiana State University graduate students in mathematics.* We have reason to believe that the figures used are approximately correct. Since almost universally is it true that the nuclei of mathematical activity are within the universities and colleges of a country, it seems fair to assume that the number of its colleges and universities, rather than its mere population, should furnish the index of expectation of the volume of its mathematical literature as well as of the number of its mathematical journals in actual circulation. However when we consider that the gross population of a nation has what we may call a mathematical potential it appears not amiss to associate the population figure with the two others named, as having a certain index value, also.

Great Britain

Population	46,189,000
Number of universities	20
Number of mathematical periodicals	5

France

Population	40,744,000
Number of universities	17
Number of mathematical periodicals	11

Germany

Population	63,178,000
Number of universities	23
Number of mathematical periodicals	15

*Miss Ruth Johnson.

Italy

Population	38,769,000
Number of universities	24
†Number of mathematical periodicals established in the last five years	5

Roumania

Population	17,393,000
Number of universities	4
†Number of mathematical periodicals established in the last five years	7

United States

Population	122,775,000
Number of universities (and colleges)	406
Number of mathematical periodicals	10

—S. T. S.

†We use here the statistics furnished by Professor Archibald alluded to above. We regret that complete statistics for Italy and Roumania are not at hand at the present time.

MISS A'S METHOD OF FINDING AREA OF A FLOOR

By M. W. COULTRAP
North Central College
Naperville, Ill.

The method the 6th grade teacher of Indiana was trying is as follows:

'You cannot multiply feet by feet. Now a floor one foot long and and one foot wide contains one square foot, for one foot times one foot gives one square foot. Now if a floor one foot long and one foot wide contains one square foot, a floor 30 feet long and one foot wide will contain 30 times one square foot which are 30 square feet; and if a floor 30 feet long and one foot wide contains 30 square feet, a floor

30 feet long and 20 feet wide will contain 20 times 30 square feet which are 600 square feet. Therefore, a floor 30 feet long and 20 feet wide contains 600 square feet."

Certainly the above is a vast display of almost meaningless jargon to the pupils. Really nonsense to them. Just about as much meaning to them as the senseless sentence (?)

"Bear me straight meandering ocean
Where the stagnant torrents flow."

Further, it revealed a remarkable lack of thinking on the part of the teacher. She said, "You cannot multiply feet by feet" and then affirmed, "One foot times one foot gives one square foot."

A thoughtful pupil would be puzzled. He surely would say to himself "If you cannot multiply 2 feet by 2 feet, or 10 feet by 12 feet, how can you multiply one foot by one foot?" Doubtless Miss C had heard somewhere that you cannot multiply feet by feet (which is correct), but there was no need of speaking of it here. It simply shows what a failure we make of it when we try to use any fact, or method, or principle before we have mastered it in every particular.

So doubtless the bright pupils, being puzzled by such contradictory statements, spent their time trying to solve that conundrum, while the teacher went on with her voluminous nonsense.

I am convinced that much of the pupil's success depends upon the teacher's presenting her work in a clear, simple and sensible way. Many teachers today are spending entirely too much time in frivolous projects and *unbaked* methods, and too little time in studious efforts to present their work clearly and concisely.

Too many presentations are much like Professor F.'s definition of Least Common Multiple.

(I quote *accurately* F.'s College Algebra, page 22.)

"Among all the common multiples of the lowest degree of two or more polynomials there is always one the greatest common divisor of whose numerical coefficients is equal to the least common multiple of the greatest common divisor of the numerical coefficients of the respective polynomials. We call this a *lowest common multiple* of the polynomials."

(The only punctuation mark is a period at the close.)

What a nonsensical combination of words. No pupil can get an *iota* of meaning from the 56 words.

A definition, if one is needed, might be something like the following: The least common multiple of two or more quantities is the least quantity divisible by each of the given quantities.

When teachers present their work in improper ways, and when university professors publish texts with such wretched definitions, how can we expect pupils to have clear conceptions of the vital principles and important facts of the subjects they are studying. Teachers must learn how to present the few vital facts and principles of their subject in clear, forceful, understandable ways, and then help the pupils to tie up the balance of the subject with these vital facts and principles.

INTRODUCING THE STUDY OF PROPORTION IN HIGH SCHOOL GEOMETRY

By F. A. RICKEY

We say that a person has a "sense of proportion" when he is quick to detect relationships and relative values. The study of proportion in high school geometry offers valuable training in this direction when conducted under the guidance of an understanding teacher. Yet it is true that many classes are never able to see anything in the mathematical proportion other than a mechanical device, a hopper into which numbers are fed by certain rules and from which the desired (perhaps only by the teacher) results are mechanically withdrawn.

If a student is introduced to the study of proportion by the information that the ratio of two numbers is the relation between them expressed by the quotient of the first divided by the second, and, furthermore, that the expression of the equality of two ratios is called a proportion, he cannot be expected to show much enthusiasm for proportions, particularly if his next task is the learning of the meaning of "antecedent", "means", "alternation", etc., in rapid order, the usual procedure in geometry texts.

The writer has found that the expressing of non-mathematical relationships in proportion form affords the student not only an

approach to the subject that he can clearly understand but also valuable practice in the sensing of relationships. A simple example is:

Feathers: chicken = (?): dog

or preferably,

$$\frac{\text{feathers}}{\text{chicken}} = \frac{(?)}{\text{dog}}$$

Examples of this kind of "proportion" can be written out and arranged somewhat in ascending difficulty as practice exercises. While this exercise does obtain the pupil's attention—he enjoys them thoroughly—it is not meant merely as a trick device to obtain attention to the proportion. It has a far greater value in giving the pupil an understanding of the expressions "is to" and "as" not obtained otherwise. He realizes the "swing" or balance of the proportion which are valuable intuitive aids in forming strictly mathematical proportions.

Other suggestive ones are:

$$\frac{\text{automobile}}{\text{wagon}} = \frac{\text{motorcycle}}{?}$$

and

$$\frac{\text{ellipse}}{\text{circle}} = \frac{\text{rectangle}}{?}$$

After this type of preparation, the pupil is ready for the formal "equality of ratios" proportion. He has a foundation upon which to build and his common sense is called upon strongly. He has the "hang of it" and the definitions of terms are more easily remembered.

The writer has found that a few days spent in introducing the study of proportions in this way are regained in the long run and that the subject is made much more vital to the student.

SOME HIGH SCHOOL FACTORS IN FRESHMAN MATHEMATICS GRADES

By R. H STEWART
Louisiana State University

This paper tells of an experiment conducted with Freshmen at the Louisiana State University intended to show, (1), the influence of the athletic coach-mathematics teacher on the pupil; and (2), the effect of additional years in high school mathematics on later grades in this subject.

In October of the 1931-32 session all Freshmen taking mathematics were asked to fill in the following questionnaire:

Name..
Graduate of......................... *High School. State*..............
I graduated in..
The football coach taught..
The basketball coach taught..
The track coach taught..
I had................................. *years high school mathematics*

A total of 547 blanks was received. Of the 547, 506 remained in classes during the first semester. Some of the 41 resigned from school, but a few dropped courses they had no hope of passing. Their names were dropped in the final calculations and do not influence the data. Had they remained in the course, the average grade would probably have been lower.

The following information was obtained from the original 547 pupils: 56 or 10.24% had less than 3 years high school mathematics; 130 or 23.76% had 3 years; 140 or 25.61% had 3.5 years; 203 or 37.11% had 4 years; and 3.29% more than 4 years. These results might have been expected except that 10.24% with less than 3 years mathematics looks a bit high. However, some of these were special students. Those with more than 4 years mathematics were usually from out-of-state or private schools. In one or two instances however, the results indicate that the student repeated some course and simply counted it an extra year. Of the 547, 153 or 27.97% were from schools where the principal coach taught mathematics alone or in combina-

tion with other subjects; 94 or 17.18% were from schools where coach taught some physical science; and 116 or 21.20% were from schools where the coach taught some social science.

Do coaches make poor mathematics teachers anyway? The table (column II) with few exceptions shows that the average grade of pupils from schools where the coach taught mathematics was very little lower than that for pupils from other schools (column III). In the totals they were nearly 4 points lower in the College of Engineering and about 2 points lower in the College of Arts and Science.

In the table the students from the different colleges are separated. Engineering students take a different course from all the others. These are all grouped as Arts and Science. The legend is:

Eng. College of Engineering.
A&S. College of Arts and Science.

Column	I	Average grade of entire class.
	II	Av. grade of pupils from coach-math. schools.
	III	Av. grade for other pupils.
	IV	Av. grade for pupils of less than 3 yrs. H. S. Math.
	V	Av. grade for pupils with 3 years H. S. Math.
	VI	Av. grade for pupils with 3.5 years H. S. Math.
	VII	Av. grade for pupils with 4 years H. S. Math.
	VIII	Av. grade for pupils above 4 years H. S. Math.

Numbers in upper half of each rectangle represent pupils considered.

The table is self explanatory. The coaching result has already been discussed. The table shows that the best grades were made by those with four years high school mathematics which strengthens the idea that those with more than four years had counted a repeat course.

	Teacher-Coach Data				High School Preparation Data											
Instructor	*I*		*II*		*III*		*IV*		*V*		*VI*		*VII*		*VIII*	
	Eng.	*A&S.*	*Eng.*	*A&S.*	*Eng.*	*A&S.*	*Eng.*	*A&S.*	*Eng.*	*A&S.*	*Eng.*	*A&S.*	*Eng.*	*A&S.*	*Eng.*	*A&S.*
A	28 79.14	**17** **80.65**	3 66.50	**3** **66.50**	25 80.66	**25** **80.66**	1 57.50	**4** **88.33**	4 67.63	**4** **66.75**	7 79.86	**6** **79.75**	15 82.17	**4** **90.13**	1 96.50	
B	48 81.06		13 77.08		35 82.54		4 84.13		15 79.63		5 78.70		19 83.42		5 76.30	
C	21 72.57		7 72.36		14 72.68		2 62.50		2 68.75		6 72.58		9 75.00		2 75.50	
D	42 81.74		14 77.48		28 83.82		2 88.25		9 81.61		13 80.38		17 81.47		1 96.50	
E	22 78.91	**109** **75.86**	5 76.90	**35** **75.04**	17 79.50	**74** **76.35**	1 96.50	**1** **57.50**	2 71.00	**24** **72.58**	12 78.88	**34** **76.41**	7 78.71	**45** **77.87**		**5** **75.40**
F	19 74.50	**79** **80.23**	6 79.83	**20** **79.93**	13 72.04	**59** **80.34**	2 58.00	**5** **71.80**	7 79.71	**21** **79.00**	7 71.71	**15** **78.50**	3 79.83	**36** **82.11**		**2** **92.50**
G		**15** **69.40**		**5** **65.60**		**10** **71.30**		**1** **71.00**		**4** **81.88**		**5** **62.90**		**4** **67.63**		**1** **57.50**
H		**33** **74.45**		**12** **72.42**		**21** **75.62**		**8** **70.50**		**9** **71.36**		**9** **78.72**		**7** **77.86**		
I		**25** **76.72**		**5** **82.20**		**20** **75.35**		**5** **67.70**		**3** **66.33**		**7** **83.86**		**10** **79.35**		
J		**48** **81.67**		**14** **78.43**		**34** **83.00**		**12** **79.50**		**12** **82.38**		**7** **85.79**		**17** **81.00**		
Totals	**180** 78.92	**326** **77.65**	48 76.17	**97** **76.46**	132 79.99	**229** **78.19**	12 75.67	**35** **74.54**	39 77.84	**77** **75.66**	50 77.63	**83** **78.85**	70 80.97	**123** **79.29**	9 80.61	**8** **77.44**

THE FIRST LESSONS IN GEOMETRY

By MRS. MARTIN L. RILEY
Critic Teacher of Mathematics, State Teachers College
Hattiesburg, Miss.

(The practical suggestions offered in this article are some of those devices and methods which seem to have proved the most successful under the direction of the Critic Teacher and student teachers in the Demonstration High School of State Teachers College during the last four years.)

The school system should provide a registration day during which pupils classify and meet classes for a few minutes to receive lists of materials to be used in the various subjects. This is especially necessary for mathematics, for the first recitations must involve pupil activity for which necessary tools must have been provided. The minimum materials for use in geometry are compasses, a protractor, a ruler (graduated in both the English and the metric system), a pencil, a notebook containing ruled and unruled paper and graph paper, and the text. To these may be added a drawing board, a T-square, a pair of dividers (similar to the compasses but usually assuring a higher degree of accuracy), thumb tacks, and a folder containing drawing paper (replacing notebook). The new Speed-up-Geometry ruler may replace several articles named. The minimum list, except the text, should be required for the first or second recitation; the text according to this plan is the least important during the first days, for it will be used very little. To assure the securing of the needed materials, the mathematics teacher in co-operation with the superintendent should see that there is a sufficient amount of materials at the local depository or store, or that the school has it to be sold to the students if the depository is not conveninently located.

If many members of the class are strangers to one another a few minutes, if the procedure is properly and naturally directed, may be spent profitably in the group's getting acquainted. (The teacher is a member of the group.) If the teacher is the only unacquainted one in the group, he may introduce himself and secure written introductions from the pupils. A plan which has been used widely and successfully is to provide each pupil with a small card, 4″x6″, convenient for filing, and ask each to answer on that card the following: Name, address, age, grade, schedule, name of parents or guardians, address of parents if the pupil is away from home, occupation of parents, how

long a member of that school, school or schools previously attended, etc. If the school has a system which provides for a complete record of the child's life as well as his educational achievements, questions should not be duplicated. The teacher should explain to the pupils that the information listed on the cards is to be strictly his; therefore, that he should like to have them tell anything which will help him to know them better, for when he knows them better he can teach them better. Some of these things might be: athletic interests, reasons for taking geometry, whether pupil thinks he will or will not like the study with reasons for statement, condition of homes, hours of outside labor, etc. If this questionnaire is well prepared, the information secured will be most invaluable, for the teacher will discover social and physical hindrances, preconceived attitudes toward the subject, etc. Although it has taken many words to describe the getting acquainted and the written introductions, it takes not more than fifteen or twenty minutes of the recitation.

Usually a big stumbling-block is preconceived notions of the difficulty of geometry; therefore, the introduction must be made so interesting and appealing that doubt of their first idea will begin to arise. Follow with an attempt to combat the notion every time it occurs. Geometry is not to be play but to be activity which the pupil can perform and it is to be presented in such a way that it challenges his best effort. Capitalize on the instincts to which geometry appeals—for instance, curiosity (there is much of the puzzle in geometry), constructions, using tools (theorems, corollaries), etc.

But how is the remainder of the period to be spent so that the beginning may be what it should be? One way which has been a success is to lead the pupils to see that they live in a land of geometry. Begin with the geometry of the class room. Do not confine the explanation to plane geometry when the concept of geometry is being developed. Discuss the discoveries made, not too technically but by summarizing the information they have previously obtained about the figures. If the study of the room is completed, ask the pupils to think of other illustrations they have seen. In this way pupils are leading which makes it necessary for the teacher to be so familiar with all forms that he can lead an interesting discussion on any discovery. The assignment is made—that is, a continuation of this quest for geometry. Pupils should be asked to name and illustrate what they find or to make an illustration or description of those things which they can not name. Make it a race to see who can find the

most. Emphasize that they have discovered most when they find forms, shapes, etc., which they cannot name.

At the next recitation the geometric forms found are named and classified, perhaps, and discussed. To meet this situation the teacher should make a broad survey of his geometric field, for he does not know what may be discovered. The winner in the quest should have honorable mention. The findings from this quest may take one period or more. Too, the teacher may lead to other discoveries—for instance, if there is a foot-ball star who has not discovered that the "pig-skin" is a geometric solid which contains several interesting curves, if the artistic girl or boy has missed the dignity of the triangle in his drawings, the similarity of figures in perspective, these pupils with special interest may be helped to see these relations.

When the discoveries furnish no further work, much use of the instruments should be begun. Likely the ruler and protractor have been used in previous work. Very likely some pupils will know some constructions which they can teach the class. Should they not be able to suggest any constructions, the teacher may produce an inscribed hexagon (previously prepared on a design, center designated). By measuring with ruler and compasses pupils find the relations of the lines, then construct the figure. Designs based on this may be shown and others found. This construction is often presented first because it is the base of many attractive designs; it is one often discovered by the pupil while merely experimenting with the compasses. There seems to be a magic about the twirl of the compasses when it produces this attractive figure. Other work which may follow is measurement in the English and the metric systems and by use of the compasses and the protractor. Other constructions which are easy and facinating are: Bisecting a line, dropping a perpendicular to a line, bisecting an angle, constructing a triangle when three sides are given, etc. Informal proofs of these constructions should be obtained through use of the measuring instruments.

All constructions accurately done should be placed in the notebook with a page allowed for each. Every construction should be named, the figure accurately constructed, "Given" and "Required" stated and the construction explained informally. This work is saved for formal proof when necessary tools have been acquired.

Construction may be made use of in the first two congruency theorems especially. Informal proofs by superposition should be all that is required at first. Exercises based on these may follow. Formal

proofs should be given a few lessons afterwards when the pupils are more familliar with the style and method of geometry.

Many axioms and postulates may be developed very meaningfully if developed concretely. (Use balances in equations, etc.) Definitions mean most if discovered as they are needed.

When formal proofs are begun, remember that it is not those concerning the congruency of triangles which are easiest. In fact to prove those triangles congruent is usually difficult because the proof seems so evident to the pupil that a long, rigorous proof in exact style seems silly and useless. Even easier than the equality of vertical angles (which is often the first formal proof), is the equality of right angles, of complements of equal angles, of supplements of equal angles, etc. All require short proofs based on axioms.

The determining of the hypothesis and the conclusion in a statement is not easy. Often these terms may be made more real if they are connected with life situations—for example: If John saves $1.00 per week for 5 weeks, he may buy a new tennis racket for the tournament. Many statements may be considered from the condition and result point of view. There should be drill in separating general statements into the "Given" and "To prove". Connections may be made with the dependent (Hypothesis) and independent (Conclusion) clauses of a complex sentence, or with the subject ("Given") and predicate ("To prove") of a simple sentence.

As much as is possible all statements used should have been proved formally or informally.

Complete independence of the text is a much better beginning than sticking closely to it. Several text-books may have guided a teacher in his preparation, but these should be put aside and real teaching done during the class period. Attacking any theorem as if it were an original is a good process. Pupils may record all findings in their notebooks; this helps them to make the findings theirs. We must summarize and drill frequently and use exercises of application generously. There should be developed an appreciation of all statements and definitions as tools for everyday work. Some texts may be used in class much more than others, but with most texts the pupils should be led to feel that their books give supplementary material, is a guide in difficult problems, and is a very ready reference. In other words, we should not let the use of the text be such that mere memorizing of a proof enables the pupil to "get by"; we should be teaching the pupils to think.

History in geometry motivates most when the history of something is introduced with the topic with which connected. At first history does not have as much appeal as does the geometry of our every-day life.

As a rule topics should be introduced where they follow most easily and most naturally—for instance, trigonometric functions after similar triangles, constructions after theorems on which based. Keep constantly in mind the general aims of mathematics and the specific aims of geometry.

In my experience nothing has proved more valuable in introductions, drills, organization, etc., than supplementary texts, work books, and mathematical publications. New and better ideas are being formed every session. We should profit not only by our schemes but also by the discoveries of others in our field.

Summarizing, may we remember to: Prepare well for the introduction of geometry, know your pupils, break up a lack of confidence in themselves, convince pupils that they know much of geometry already and that they live in a geometrical world, lead a group of discoverers, engage much of pupil activity, present the easiest first, correlate with other subjects, and so organize the work that there are immediate uses for everything learned, with one topic leading naturally to another. In the words of Dr. Charles McMurry, George Peabody College, "If something is worth teaching, it is worth sweating blood to teach."; therefore, let us resolve to start the geometry session right by giving an introduction that is our best.

THE WORD TRANSPOSE IN ALGEBRA

By W. V. PARKER
Mississippi Woman's College
Hattiesburg, Miss.

At the beginning of a freshman course in college a few years ago, I wrote the equation $3x=10$ on the blackboard and asked for the value of x. I received four different answers in the order (1) $x=7$, (2) $x=13$, (3) $x=30$, (4) $x=10/3$. The reason for this is, I believe, due to to the fact that at some time the students had got the idea of carrying a number across the equation sign without realizing what algebraic operation they were performing. This seems to emphasize the impor-

tance of keeping the fundamental operations of addition, subtraction, multiplication and division before the students all the time. It also raises a question as to the advisability of using the word transpose at all.

In solving the equation $x-2=5$, would it not be better to say "add 2 to both members getting $x=7$" rather than "transpose the 2 getting $x=7$?" In reality the first operation is the one which we are performing. It is impossible to carry the 2 across the equation sign, yet we find the word transpose in most text books and in many cases they do not define what they mean by the term. If we stick to the fundamental operations, I believe that many errors of the type noted above will be avoided.

SIMPLY DERIVED FORMULAE FOR DETERMINING THE THIRD PART OF AN ARBITRARY ANGLE

By REV. F. M. KENNY, D. D.
Malone, N. Y.

With OA as a radius and O as a center describe a circle intersecting OB′ at B.

Produce AO to C, BO to D.

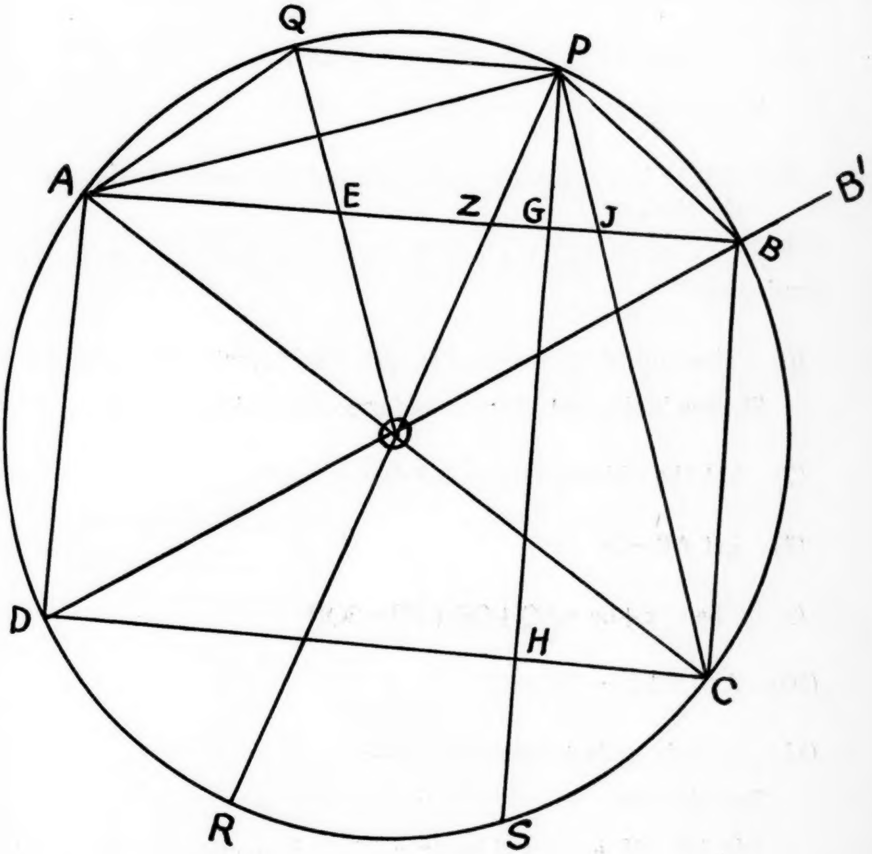
Assume angle AOB trisected by line OP (P being at the intersection of OP and the circle). Produce OP to R. Draw OQ similarly, E being intersection.

Join BC, AD, DC, AP, PC, PD, PB, AB.

Let fall a perpendicular from P to G (on AB), intersecting DC at H and the circle at S. (Note: $CS=BP$, $CH=BG$).

Letter the intersection of PC and AB "J", and the intersection of PR and AB "Z".

Proof



(1) Triangles AGP and PGJ are similar.

Angle APC is rt. (inscribed in semi-circle). Perpendicular to hyp. of rt. triangle divides it into two similar triangles.

(2) Triangle PGJ and BCJ are similar.

Angle ABC is rt. (inscribed in semi-circle). Rt. angle and acute angle of each are equal.

(3) Triangle BPD is similar to AGP (and therefore to PGJ)

Angle BPD is rt (inscribed in semi-circle). Angles A and D measured by $\frac{1}{2}$ same arc.

(4) Angles CPS, PAB, PCB, PDB are equal

Measured by equal arcs.

(5) ABCD is a rectangle with diagonals AC and BD diameters of the circle.

Angles ABC, BCD, CDA, DAB are rt. angles (inscribed in a semi-circle.)

(6) The sum of the chords AQ, QP, PB is greater than chord AB

St. line is shortest distance between two points.

(7) Let the difference be called 6a

(8) Let $AB = 3x$

(9) Then $3x + 6a = AQ + QP + PB = 3QP$

(10) And $x + 2a = QP = BP$

(11) Also $AG = 2x + a$ and $BG = x - a$

Substitution.

AQ, QP, BP are equal because QP or its equivalent EJ, or $x+2a$, must take an "a" from each side in addition to the central third of AB which is "x".

(12) In the similar triangles

AGP and PGJ:

$$AG:GP::GP:GJ$$

Corresponding sides of similar figures are proportional.

$$AG \times GJ = GP^2 = PB^2 - BG^2$$

Product of means = product of extremes.

$$PB^2 - BG^2 = (x+2a)^2 - (x-a)^2$$

Side of rt triangle squared = hyp^2 − (other side)^2

$$(x-a)^2 = 3a(2x+a)$$

Substitution.

(13) or $AG \times GJ = 3a(2x+a)$

Perpendicular to hyp. of rt. triangle is mean proportional between segments. Product means = product of extremes.

(14) But $AG = 2x+a$

See (11).

(15) therefore $GJ = 3a$

Equals divided by equals.

(16) Call the ratio $\dfrac{AG}{GJ} = r^2$

(17) So $GJ = 3a$

$$GP = 3ar$$

$$AG = 3ar^2$$

See (15).

$3ar^2 : GP :: GP : 3a = 3ar$ (Product means = product extremes)

$$\frac{AG}{3a} = r^2,\ AG = 3ar^2$$

(18) but $2x+a = 3ar^2$

Substituting.

(19) $$x = \frac{3ar^2 - a}{2} = \frac{a}{2}(3r^2-1)$$

(20) $$x + 2a = \frac{a}{2}(3r^2+3)$$

$$x - a = \frac{a}{2}(3r^2-3)$$

(21) In triangle JGP the sides are

$$GJ = 3a, \; GP = 3ar$$

$$JP = 3a\sqrt{r^2+1}$$

Substitution and rules for rt triangle.

(22) In similar triangle JBC:

$$JB = x - 4a$$

Because $BG = x - a$ and $GJ = 3a$.

(23) So, as $JB = x - 4a$

$$BC = (x - 4a)r$$

Corresponding sides of similar triangles are proportional.

(24) In the other similar triangle BPD:

$$BP = x + 2a$$

See (10).

$$\text{So } BD = (x+2a)\sqrt{r^2+1}$$

Corresponding sides of similar figures are propor.

$$x + 2a = \frac{3a}{2}(r^2+1)$$

See (20).

$$\text{Hence BD} = \frac{3a}{2}(r^2+1)\sqrt{r^2+1}$$

Substitution.

(25) So the four important lines are:

$$1)\quad AB = 3x = \frac{3a}{2}(3r^2-1)$$

See (19).

$$2)\quad BC = \frac{3a}{2}(r^2-3)r$$

See 24a (below).

$$3)\quad BD = \frac{3a}{2}(r^2+1)\sqrt{r^2+1}$$

See (24).

$$4)\quad BZ = \frac{3a}{2}(r^2+1)$$

See 24a (below). GJ = GZ because triangle JGP = ZGP.

(24a)
$$x-4a = \frac{a}{2}(3r^2-1-8)$$

$$= \frac{3a}{2}(r^2-3)$$

See (20)

(26) $$\text{So } GZ = 3a, \; BG = x - a$$

Substitution.

$$\text{and } BZ = x + 2a = \frac{3a}{2}(r^2+1)$$

See (24a).

(27) Therefore we have *proportional* values as follows:

$$AB = 3r^2 - 1$$

$$BC = (r^2-3)r$$

$$BD = \sqrt{(r^2+1)}\,(r^2+1)$$

$$BZ = r^2+1$$

$$\text{Common factor } \frac{3a}{2} \text{ removed.}$$

[**Editorial Remark**: Dr. Kenny's development in so simple a manner of the formula r^2+1 for the portion of the chord subtending AOB/3 should be of more than passing interest to one concerned with the general angle-trisection problem. No one is more aware than Dr. Kenny that, except in special cases, this formula is not constructible by ruler and compasses. Nevertheless, it is an interesting fact that his set of four formulae, designated (27), are effective for identifying both the angles which are, and those which are not, trisectible by ruler and compasses. For instance, for every value of r that is rational, the four lengths AB, BC, BD, BZ are constructible in this manner and the angle AOB is trisectible since its third part is then determined by the straight line OZ. Like conclusions follow if r is any rational square root or any rational function, or square root irrational function, of rational square roots and rational numbers. But, for all other values of r, the angle AOB is *not* trisectible by ruler and compasses, inasmuch as none of the formulae of (27) are then constructible in this manner. For example, if $r = \sqrt[3]{31}$, or $\sqrt[5]{67}$, though theoretically, the third part of AOB′ is determined, mere ruler and compasses cannot construct it, since r^2+1 cannot in this case be constructed.

It is evident that, while for a given r a unique angle AOB′ results, its measure in degrees is not necessary in order to determine its trisectibility or non-trisectibility.

But in the classic problem of trisection we assume an arbitrary angle, and that this angle is the *given* thing, not r.

In such case the cosine of $\frac{1}{2}(180° - AOB')$ would be $\dfrac{3r^2-1}{(r^2+1)\sqrt{r^2+1}}$. Denoting this

angle by B, we should have, $\cos B = \dfrac{3r^2-1}{(r^2+1)\sqrt{r^2+1}}$. Thus the determination of whether or not r is subject to a ruler—and—compass construction and hence, whether or not AOB′ is trisectible, rests with the absolute value of cos B, and the solution for r of the resulting equation. If B should have such a value in degrees that cos B could only be approximately expressed, then a solution of the corresponding equation in r could only be approximated and the angle-trisection would only be *approximate*. On the other hand, if cos B = a rational number, say c, we should have $c = \dfrac{3r^2-1}{(r^2+1)\sqrt{r^2+1}}$. Letting r = u, for convenience, clearing and squaring, this would become

$$c^2u^3+u^2(3c^2-9)+u(6+3c^2)+c^2-1=0$$

If c should be such a value that this cubic equation in u $(=r^2)$ has a rational root u, then will u $(=r^2)$, be constructible and AOB will be trisectible by ruler and compasses. But if the equation should have no rational root, then would AOB not be trisectible in this manner.—S. T. S.]

THE EFFECTIVE RATE CORRESPONDING TO A DISCOUNT OF 1% PER MONTH

By IRBY C. NICHOLS
Louisiana State University

[*Address delivered March 11, 1932, before the Louisiana - Mississippi Section of the Mathematical Association of America at its annual meeting.*]

The following article is written in response to frequent requests of business men whose labors lie within financial fields. For teachers, its chief claim to cleverness is in its calling attention to another nice use of the binomial theorem, whose value is too often overlooked or discounted by mathematics teachers, both in the High Schools and in the Colleges.

The problem in mind is this: *A, B, C, and D, borrow $100.00 each from a loan company, they pay as interest in advance $10.00, $12.00, $15.00 and $18.00 respectively; further, they sign notes whereby they agree to repay their loans of $100.00 each in equal monthly installments over periods of 10, 12, 15, and 18 months respectively, the first installment to be paid, in each case, one month from date of loan. What effective rate of interest does each borrower pay?*

Note that each borrower pays a *flat discount of one per cent per month* during the life of his loan, a schedule of interest charges familiar

to *installment* purchasers. It may seem that A, B, C and D should pay the same *effective* rate of interest also, but they do not: *lengthening* the time of payments *increases* the *effective* rate of interest. To simplify procedure, let us study A's case first.

First method of solution: Notice that A is to pay $10.00 per month for 10 months, a total of $100.00, although the cash value of his loan is only $90.00, this being the amount which he received on his note, since he gave back $10.00 as interest paid in advance. A common method for handling a schedule of this kind is to use the *average* life of the loan with the traditional ***bank discount*** rule. The first payment of $10.00 is to be made in one month, the second in two months, the third in three months, and so on for ten months. The arithmetic average is therefore five and a half months. The payments are equal in size. So it is common to assume that the whole loan of $100.00 may be quite equitably paid at exactly 5.5 months after the date of the note; and that the sum of the discounts on the several monthly payments is equivalent to the discount on the whole, which is $10.00.

Hence we have this equation: $100.00 \times i \times \dfrac{5.5}{12} = 10.00$, where i is the effective rate of interest sought, and 5.5/12 is the *average* time expressed in years. Solving this equation, $i = 21.82\%$.

It is a familiar fact that this method does not yield results that are absolutely correct, particularly when the time interval is long; yet this method, being easily understood and readily applied, is frequently used.

Second Method. If the *average* time, as obtained above, be used with the method known as *true discount*, the results obtained will be more accurate. The cash value of A's loan is $90.00, since he returned $10.00 of his loan of $100.00 as interest payable in advance. Therefore $100/(1+5.5/12i) = 90$. Whence $i = 24.24\%$

This method is also of common practice, but it, too, has objections: neither of these first two methods contemplate a *monthly turnover* of the capital invested; also, both of them use the *average* time.

Third Method. Invoking the aid of logarithms, a method using the same *average* time employed above, but involving a *monthly turnover* of the capital is found in a direct use of our familiar formula

$A=P(1+i)^n$, where $A=\$100.00$, $P=\$90.00$, $n=5.5/12$ years, and i is the effective rate sought. Substituting these respective values in our given formula, and, applying logarithms, $\log(1+i)=\dfrac{12}{5.5}(\log 10-\log 9)=.099834$; whence $1+i=1.2584$, and $i=25.84\%$.

This method has only the objection of the use of the *average* life of the installment payment, instead of the *actual* life of each respectively.

Fourth Method. Assuming a monthly turnover of the capital invested, the present value of each monthly payment may be found by a repeated application of the familiar formula of the third method above. Then the sum of these present values must equal the present value of the whole, that is \$90.00. Hence the equation $90=10/(1+i)^{1/12}+10/(1+i)^{2/12} \dots 10/(1+i)^{10/12}=10(1+i^{-1/12}+10(1+i)^{-2/12}+10(1+i)^{-3/12}+\dots\dots+10(1+i)^{-10/12}$, the exponents of $(1+i)$ being the respective time intervals expressed in years, and i being the effective rate sought. The right hand member of this equation is a geometric series whose first term a is $10(1+i)^{-1/12}$, whose last term l is $10(1+i)^{-10/12}$, and whose ratio r is $(1+i)^{-1/12}$. The formula for finding the sum of a geometric series is $(rl-a)/(r-1)$. Applying this formula here and simplifying the result, we obtain $90(1+i)^{11/12}-100(1+i)^{10/12}+10=0$. Expanding the terms of this equation by the binomial theorem and dropping all terms containing i with an exponent equal to or greater than 4, we have $1513i^3-3636i^2+864i=0$. By factoring, we have $i=0$ and $1513i^2-3636i+864=0$. From this last equation, $i=2.135$ or $.267$. Only the last value of i is compatible with the practical conditions of the problem assigned. Hence $i=26.7\%$ is the value sought.

The first two decimal places of this result are correct; the third place may be off a little; but it may be found accurately by a repetition of this method, by taking care first to replace i in the equation $90(1+i)^{11/12}-100(1+i)^{10/12}+10=0$ with $i'+.267$, thus obtaining i' and adding it to .267 (or subtracting it as the case may require); and then repeating the process as many times as may be needed to obtain the value of i correct to the desired number of decimal places.

The Cases of B, C, and D. By methods exactly as in the case of A, results may be found for B, C, and D. This information is shown in the table below.

The table shows also results for F, who borrowed \$100.00 payable in 10 equal monthly payments but at a flat discount of only 8%. This case does not come under the case of "one per cent per month," but it is common in practice, and is given here as a matter of convenience to those wishing a complete schedule for their files. It was solved in a previous issue of the News Letter, vol. 3, No. 7, March, 1929.

Nominal Rate	*Equation Used with Fourth Method*	*First Approx.*	*Second Approx.*	*Third Approx.*	*Effective Rate*
10%, 10 Mos.	$90(1+i)^{11/12} - 100(1+i)^{10/12} + 10 = 0$	21.82	24.24	25.84	26.7
12%, 12 Mos.	$88(1+i)^{13/12} - 96\frac{1}{3}(1+i)^{12/12} + 8\frac{1}{3} = 0$	22.15	25.17	26.61	27.5
15%, 15 Mos.	$85(1+i)^{16/12} - 91\frac{2}{3}(1+i)^{15/12} + 6\frac{2}{3} = 0$	22.50	26.47	27.60	28.6
18%, 18 Mos.	$82(1+i)^{19/12} - 87\frac{5}{9}(1+i)^{18/12} + 5\frac{5}{9} = 0$	22.74	27.73	28.49	29.4
8%, 10 Mos.	$92(1+i)^{11/12} - 102(1+i)^{10/12} + 10 = 0$	17.45	19.75	19.95	20,2

A study of the above table shows these facts: (1) The *bank discount* method, the first method, using an *average* time interval gives results differing little from each other, but differing much from the correct results; (2) the *true discount* method, the second method, with an *average* time interval, gives results varying more from each other than the results of the first method, and coming closer to the true results. To the business man, unskilled in the use of logarithms or in the binomial theorem, this method is recommended. Not even Senior High School training is necessary to handle it. (3) A knowledge of logarithms combined with the use of the *average* time, furnishes the short and fairly accurate results of the third method. (4) The binomial theorem furnishes the method that is nice, convenient and accurate. It will be appreciated by all persons engaged in the business of handling loan paper. This method should not be beyond the ability of any Senior High School boy or girl.

The equations involved in the fourth method are shown in the table as a matter of interesting comparison: The co-efficient of the first term is the cash value of the loan, the last term is the monthly payment, and the co-efficient of the middle term, with the signed changed, is the sum of the cash value and the monthly payment. Further, the exponent of $(1+i)$ in the middle term is the life of the loan expressed in years, and the exponent of $(1+i)$ in the first term is the life of the loan increased one month and then expressed in years. Herein we have a rule for writing equations for other rates without the tedious labor of deriving such equations in detail.

ON MOMENT OF INERTIA

By W. PAUL WEBBER
Louisiana State University

In one book we read that moment of inertia is a mathematical expression which has such and such properties, and can be used for calculating the bending stresses in beams. In another book we can read that moment of inertia is a physical quantity which performs the same role in rotating bodies that ordinary inertia plays in the motion of bodies in linear displacement. Again we encounter a quantity of similar mathematical form when we calculate the standard deviation of a set of statistical data. Now, is moment of inertia all of these varied entities, or is it none of them, or is it one of them, and by virtue of form happens to be a name applied to two or more of them for convenience?

It requires the application of force to change the magnitude or the direction of motion of any mass, or to set it in motion if at rest. The application of force in such cases seems to call out a resistance just equal to the force applied. This so called resistance is called inertia. The measure of this resistance is

$$(1) \quad f = ma,$$

where m is the mass, and a is the acceleration produced by the force f.

One probably first meets the term moment in connection with the study of levers. A lever is in effect a rigid bar hinged at some point, and is to have a force applied at one end to move a body, or balance a force applied at some other point of the bar. The effectiveness of the force acting on the lever is measured by the magnitude of the force, or its component perpendicular to the lever, multiplied by the distance of the point of application of the force from the hinge, or fulcrum of the lever. Thus we have for moment of a force

$$(2) \quad M = fr,$$

where M is the moment of the force f at a distance r from the fulcrum. M is called the moment of the force. In producing rotation of a body about an axis the measure of the effectiveness of the force employed is measured by its moment with respect to the axis about which the body is to rotate. It is found that there is a simple relation between

the resistance of a body to linear motion and its resistance to motion of rotation. Consider a mass m constrained to move in a circle of radius r about a center O. The linear acceleration of the mass is related to the force by (1). Now by a suitable choice of angular units the linear acceleration in the circle is equivalent to $a = r\alpha$, where α is the angular acceleration. Now if we multiply (1) by r on both sides we have

$$(3) \quad fr = mar.$$

Now if we transform the linear acceleration to its equivalent value in angular acceleration at distance r from the center we obtain

$$(4) \quad fr = mr^2.$$

Now by definition this is to mean

$$(5) \quad fr = I\alpha,$$

where I is the moment of inertia of the mass m about the center O. It is thus seen that moment of inertia is a physical quantity that has properties in relation to rotating bodies that are analogous to ordinary inertia in relation to linear motion.

The idea of moment seems to have its origin in the laws of the lever. But even so, it appears to be only a special case, namely, moment of force. The underlying correlating idea is that of something acting or being applied at a distance from some center of position, or motion. If some mass is used instead of force we have mr as the moment of mass at distance r from the center. If instead of force or mass we have some statistical quantity q whose deviation from the median, or the mean, is d we have the moment of q as qd. In calculating standard deviation we have qd^2.

All these apparently different variations of the same idea may be harmonized by being considered particular cases of a general notion of moment, defined in some such way as the following: The moment of any quantity located at distance, either in space, or in deviation from some standard number, is the product of the magnitude of the quantity by its distance from the standard position, or number.

Under this idea moment of inertia becomes what is sometimes called the second moment of mass, that is mr^2. The quantity employed

in standard deviation may be called the second moment of frequency. The quantity that engineers call moment of inertia, in calculating the strength of beams, would be more properly called second moment of area of the cross section of the beam. The ordinary moment of a force would be the first moment of the force and the product of the frequency of a statistical quantity by its deviation from the median, or mean, would be the first moment of the frequency. It appears that a slight change in the usual phraseology might remove some serious difficulties in the way of the understanding of the beginner in any of these studies.

PROBLEM DEPARTMENT

Edited by
T. A. BICKERSTAFF
University, Miss.

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and solve problems here proposed.

Problems and solutions will be credited to their authors.

Send all communications about problems to T. A. Bickerstaff, University, Miss.

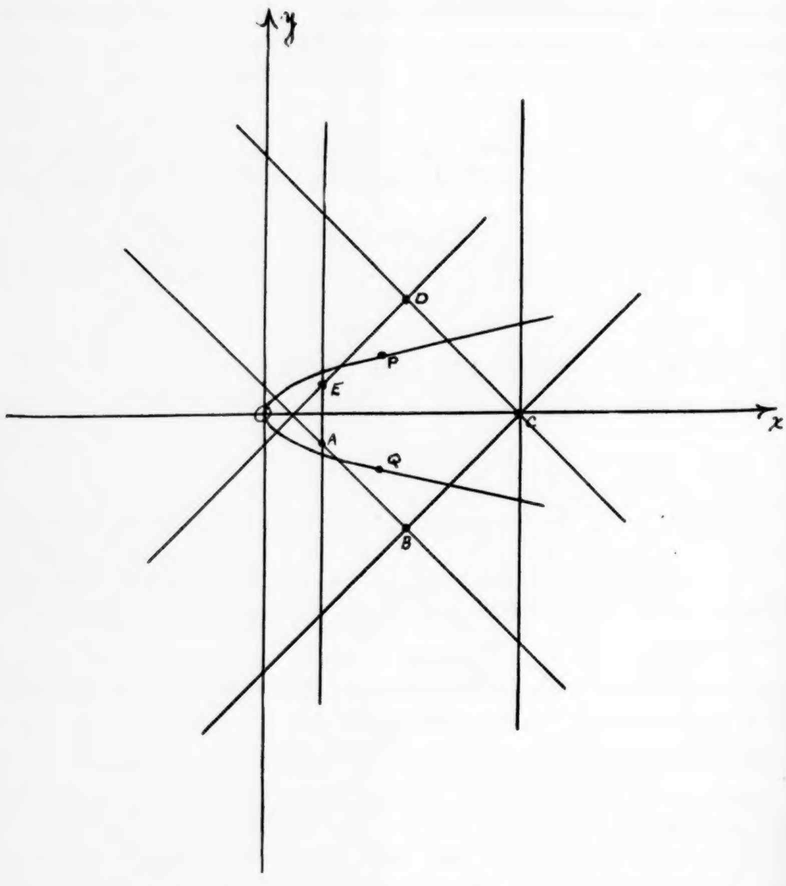
Solutions

No. 10. Proposed by T. A. Bickerstaff, University, Miss:

The Digits of a three-digit number form an arithmetic progression. The digits of 10 less than the number form a harmonic progression. Find the number.

Note: Solutions for this problem have already been published. The following different approach is found to be very interesting. It follows:

Solved by William E. Byrne, Virginia Military Institute, Lexington, Va.



Let the required digits be $x-y$, x, $x+y$. This gives us three inequalities:

(1) $1 \leqq x-y \leqq 9$ (since the required number has three digits, the first one of which is not zero).

(2) $0 \leqq x \leqq 9$

(3) $0 \leqq x+y \leqq 9$

Evidently x and y must also be integers.

The number 10 less than the required number has the digits $x-y$, $x-1$, $2+y$. This gives the additonal inequality

(4) $0 \leq x-1 \leq 9$

From the equation of the problem it follows that

(5) $(x^2-y^2)(x-1) \neq 0$

(6) $y^2 = x$

The problem is to determine integral solutions for x and y obeying the five inequalities (1)–(5) and the equality (6).

In view of (5) we modify (3) to give

(3′) $1 \leq x+y \leq 9$

and also we modify (4) to give

(4′) $1 \leq x-1 \leq 9$

or, what amounts to the same thing,

(4″) $2 \leq x \leq 10$

(2) and (4″) combine to give

(7) $2 \leq x \leq 9$

The final system to be satisfied is then made up of the inequalities (1), (3′), (7) and the equality (6). This means that the values (x,y) desired must correspond to points within the polygon ABCDE which lie on the graph of the parabola (6) with the further restriction that x and y must be integral. Inspection of the graphs shows that there are two such points P(4,2) and Q(4, −2). The solution corresponding to the point P of the graph is the number 246 and the solution corresponding to the point Q is the number 642.

No. 12. Proposed by T. A. Bickerstaff, University, Miss.

A pendulum swings east and west in a wind from the west so that each swing westward is $^7/_{10}$ as long as the preceding eastward swing and each swing eastward is $^{13}/_{10}$ as long as the preceding westward swing. Find the distance traversed by the pendulum in coming to rest if the first swing counted is eastward and equals 100 centimeters, assuming that the time of each swing is proportional to the length.

Solved by J. C. Currie, Hattiesburg, Mississippi, and the proposer.

The situation described in this problem applies the following series: $a+ar+arR+ar^2R+a\ r^2R^2+ar^3R^2+\ldots\ldots$ where r and R are different and $|rR|$ is less than 1. This may be written,

$a(1+r)\ (1+rR+r^2R^2+r^3R^3+\ldots\ldots)$ which is recognized as a geometric series. Therefore, in our problem,

$$S_n = \frac{100(1+{}^7/_{10})}{1-{}^7/_{10}\cdot{}^{13}/_{10}}$$

$$= \frac{17000}{9}$$

No. 16. As a result of a typographical error, this problem as printed read, Find y^2 when $y=\sqrt{x-\sqrt{x-x\sqrt{-\sqrt{x}}}}\ldots$ ad. inf.

This error is regretted. It is corrected elsewhere as problem number 10. The following solutions of the problem as printed were received:

A— Solved by J. T. Fairchild, Ada, Ohio.

Squaring, we have, $y^2=x-y$

Then, $$y = \frac{-1\pm\sqrt{4x+1}}{2}$$

and, $$y^2 = \left(\frac{-1 \pm \sqrt{4x+1}}{2} \right)^2$$

B. Solved by William E. Byrne, Virginia Military Institute, Lexington, Va.

Solution of problem 16, Mathematics News Letter, January, 1932.

A heuristic solution is given right away if we square the given expression, without inquiry as to whether or not we have a right to do so. We find

$$y^2 = x - \sqrt{x - \sqrt{x -}} \ldots \qquad = x - y$$

so we might conclude that y is a root of that equation. But which root do we take and what is the range of validity?

The problem is equivalent to determining the limit of the sequence of functions $y_1, y_2, \ldots, y_n, \ldots$ defined as follows:

$$y_1 = f_1(x) = \sqrt{x}$$

$$y_2 = f_2(x) = \sqrt{x - y_1}$$

$$\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$

$$y_{n+} = f_{n+1}(x) = \sqrt{x - y_n}$$

$$\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$

$f_1(x)$ is defined for $x \nless 0$. $f_2(x)$, however, is defined only for $x \nless 1$; hence all $f_n(x)$, $n \nless 1$, are defined only for $x \nless 1$. Furthermore, there is no unique limit function for $x = 1$ since

$$f_1(1) = 1,\ f_3(1) = 1,\ \ldots,\ f_{2n+1}(1) = 1,\ \ldots$$

$$f_2(1) = 0,\ f_4(1) = 0,\ \ldots,\ f_{2n}(1) = 0,\ \ldots$$

For convenience of notation we introduce $y_0 = f_0(x) = 0$. In what follows we shall assume $x > 1$.

We note that

$$y_p - y_q = \frac{y_p^2 - y_q^2}{y_p + y_q} = \frac{(x - y_{p-1}) - (x - y_{q-1})}{y_p + y_q}$$

$$(1) \quad y_p - y_q = \frac{y_{q-1} - y_{p-1}}{y_p + y_q}, \qquad (p > 0, q > 0)$$

and as a particular consequence of (1) we have

$$(2) \quad y_{p+1} - y_p = \frac{y_{p-1} - y_p}{y_p + y_{p+1}}$$

and likewise we have

$$(3) \quad y_{p+2} - y_p = \frac{y_{p-1} - y_{p+1}}{y_p + y_{p+2}}$$

By successive application of these formulas and by mathematical iinduction we conclude that the y's with odd subscripts constitute a monotonic decreasing sequence and that the y's with even subscripts constitute a monotonic increasing sequence. Hence we see that since $y_{2n+1} > 0$ and $y_{2n+1} < y_{2n-1}$ that

$$\lim_{n \to \infty} f_{2n+1}(x)$$

exists. For similar reasons, $f_{2n}(x)$ remaining bounded by $f_1(x)$,

$$\lim_{n \to \infty} f_{2n}(x)$$

exists. From inequality (2) we see that

$$f_{2n}(x) < f_{2n+1}(x)$$

so that

$$(4) \qquad \lim_{n \to \infty} f_{2n}(x) \not> \lim_{n \to \infty} f_{2n+1}(x)$$

If the equality sign holds in (4) we should have

$$(5) \qquad \lim_{n \to \infty} y_{2n+1} = y = \lim_{n \to \infty} \sqrt{x - y_{2n}} = x - y$$

This gives

$$(6) \qquad y^2 = x - y$$

with the restrictions

$$y \nless 0$$

$$x - y \nless 0$$

The solutions of (6) are

$$y = -\tfrac{1}{2} + \sqrt{x + \tfrac{1}{4}}$$

$$y = -\tfrac{1}{2} - \sqrt{x + \tfrac{1}{4}}$$

$y = -\frac{1}{2} + \sqrt{x + \frac{1}{4}}$ is the required solution, satisfying the restrictions mentioned above. We must show, however, that we have a right to use the equality sigh and to reject the inequality sign.

We have shown that $\lim_{n \to \infty} y_{2n}$ and $\lim_{n \to \infty} y_{2n+1}$ exist. But

$$y_{p+2} = \sqrt{x - y_{p+1}} = \sqrt{x - \sqrt{x - y_p}}$$

If p is odd and $\lim_{p \to \infty} y_p = y^*$, or if p is even and $\lim_{p \to \infty} y_p = \bar{y}$ then y^* and $\bar{y}$ are roots of the equation

$$u = \sqrt{x - \sqrt{x - u}}$$

or

$$u^2 = x - \sqrt{x-u}$$

$$x - u^2 = \sqrt{x-u}$$

$$u^4 - 2u^2x + x^2 = x - u$$

(7) $$u^4 - 2xu^2 + u + x^2 - x = 0$$

By substitution we find that

$$u = \sqrt{x+\tfrac{1}{4}} \quad -\tfrac{1}{2}$$

and

$$u = -\sqrt{x+\tfrac{1}{2}} \; -\tfrac{1}{2}$$

are roots of (7). Hence

(8) $$u^4 - 2xu^2 + u + x^2 - x \equiv (u^2 + u - x)(u^2 - u - x + 1)$$

We are allowed to use only solutions of (7) which satisfy for $x > 1$ the inequalities (equivalence of equations):

(9) $$\begin{cases} u > 0 \\ x - u > 0 \\ x - u^2 > 0 \end{cases}$$

The only solution satisfying these requirements is

$$u = \sqrt{x+\tfrac{1}{4}} \quad -\tfrac{1}{2}$$

Hence we use the equality sign in (4).

Conclusion: The given expression

$$y = \lim_{n\to\infty} y_n$$

is defined for $x > 1$.

$$y = \sqrt{x + \tfrac{1}{4}} \quad - \tfrac{1}{2} = f(x)$$

Hence the required result is

$$y^2 = x + \tfrac{1}{2} - \sqrt{x + \tfrac{1}{4}}$$

We note that

$$f(1) = \tfrac{1}{2}(\sqrt{5} - 1)$$

but that

$$\lim_{n \to \infty} f_n(1)$$

has no meaning.

Late Solutions

No. 14. Solved by Janet McDonald, Mississippi Synodical College, Holly Springs, Mississippi, and by Elmer E. Pratt, Jennings, Louisiana.

Problems for Solution

No. 17. Proposed by E. C. Kennedy, College of Mines and Metallurgy University of Texas, El Paso, Texas.

Find y', that is, $\dfrac{dy}{dx}$ when

$$y = \sqrt{x - \sqrt{x - \sqrt{x - \sqrt{x -}}}} \quad \ldots \text{ ad. inf.}$$

No. 18. Proposed by T. A. Bickerstaff, University, Mississippi.

Eliminate θ from the system,

$$x = \cos 2\theta + 2\cos\theta$$

$$y = \sin 2\theta - 2\sin\theta$$

No. 19. Proposed by William E. Bryne, Virginia Military Institute, Lexington, Va.

The function $f(\theta) = \dfrac{\sin 3\theta}{\cos\theta}$

is 0 for $\theta = 0$ and for $\theta = \dfrac{\Pi}{3}$. In the open interval $\left(0, \dfrac{\Pi}{3}\right)$ $f(\theta) > 0$, and since it is continuous it must have at least one maximum. Show that there is only one maximum and find its value in its simplest form.

No. 20. Proposed by J. T. Fairchild, Ada Ohio.

Integrate the following:

$$S = \int [2a^2+2c^2-2a(a^2+c^2)^{1/2}+(a^2+c^2)\theta^2]^{1/2} d\theta$$

No. 21. Proposed by T. A. Bickerstaff, University, Mississippi.

The angles of a plane triangle form a geometric progression whose common ratio is 3. Show that the ratio of the perimeter to the smallest side is

$$8 \sin \frac{10\Pi}{26} \sin \frac{11\Pi}{26} \sin \frac{12\Pi}{26}$$